Energy Dissipation in Turbulence

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Numerical Analysis and Predictability of Fluid Motion
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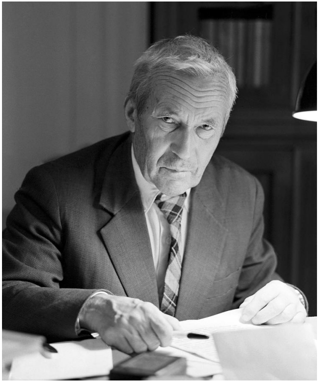


Kolmogorov: *Predicting turbulence statistics correctly requires predicting energy dissipation rate correctly.*

Introduction

- The Smagorinsky model (SM)

$$u_t + u \cdot \nabla u - \nu \Delta u + \nabla p - \nabla \cdot ((C_s \delta)^2 |\nabla u| \nabla u) = f(x) \quad \text{in } \Omega,$$

$$\nabla \cdot u = 0 \quad \text{in } \Omega.$$

- Time-averaged Energy Dissipation rate

$$\langle \varepsilon(u) \rangle = \limsup_{T \to \infty} \frac{1}{T} \int_0^T \left(\frac{1}{|\Omega|} \int_\Omega \nu |\nabla u|^2 + (c_s \delta)^2 |\nabla u|^3 dx \right) dt.$$

- SM over-dissipates.
- **Does SM with Damping functions over-dissipate ?**

NSE vs SM

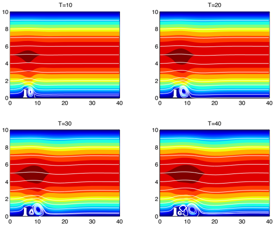


Figure: NSE, eddies are shed and roll down channel

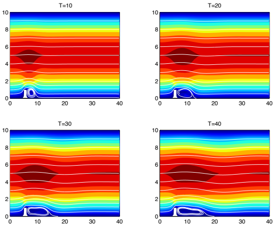


Figure: SM, flow reaches equilibrium quickly!

History and Motivation

- Boussinesq 1877; *Turbulent fluctuations are dissipative in the mean*.
- $\langle \varepsilon(u) \rangle \simeq \frac{U^3}{L}$.
- Doering and Constantin 1992; in NSE for Shear flow.

$$\langle \varepsilon(u) \rangle \leq C \frac{U^3}{L}.$$

- Layton 2002; SM for Shear flow over dissipates

$$\langle \varepsilon_s(u) \rangle \simeq [1 + C_s^2 (\frac{\delta}{L})^2 (1 + \mathcal{R}e)^2] \frac{U^3}{L}.$$

- Layton 2016; SM with under periodic B.C

$$\langle \varepsilon(u) \rangle \simeq \frac{U^3}{L}.$$

Practical Solutions

In practice, model refinements aim at reducing model dissipation by

- Structural sensors,
- Dynamic parameter selection,
- Accentuation technique,
  and
- Damping functions.

van Driest Damping function

- $(C_s\delta)^2|\nabla u|$; large near the walls $\Rightarrow$ over-dissipation.
- Van Driest; $C_s = C_s(x) \to 0$ as $x \to$ wall.
- Numerical simulation; SM + van Driest performs better.

Smagorinsky + Damping function

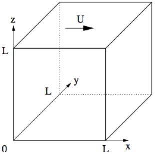
Let $\Omega = [0, L]^3$. Find (u, p) satisfying

$$u_t + u \cdot \nabla u - \nu \Delta u + \nabla p - \nabla \cdot (\beta(x)(C_s\delta)^2 |\nabla u| \nabla u) = 0, \qquad (1)$$

$$\nabla \cdot u = 0.$$

Boundary conditions

- Periodic boundary conditions in the x and y directions,
- $u(x, y, 0, t) = (0, 0, 0)^\top$ and $u(x, y, L, t) = (U, 0, 0)^\top$. (2)



Results

Theorem

Let $\beta(x, y, z) : [0, L]^3 \longrightarrow \mathbb{R}$ to be $\beta(x, y, z) = (\frac{z}{L})^\alpha (1 - \frac{z}{L})^\alpha$, then for equation (1) and the boundary conditions (2) with $\alpha \geq 2$, we have

$$\langle \varepsilon(u) \rangle \leq C_1 \frac{U^3}{L} + C_2 (\frac{C_s \delta}{L})^2 \ \frac{U^3}{L},$$

where C_1 and C_2 are positive and independent of viscosity.

- The estimate is consistent with phenomenology and the rate proven for NSE.

Current Work

- Energy dissipation of discrete model in a fixed grid.
- Fluctuating shear velocity.

Thank you !